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STATISTICAL TREATMENT OF ARAMID/EPOXY STRESS-RUPTURE DATA

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The purpose of this report is to provide statistically derived tabular design data to predict the estimated stress-rupture life of unidirectional aramid/epoxy composites subjected to sustained tensile loading. The maximum likelihood estimated life (MLEL) is tabulated for various stress levels of the fiber stress and quantile levels. The estimated lifetimes were calculated from accelerated stress-rupture tests of T.T. Chiao, who used uniaxial strand samples; therefore, the estimated lifetimes reported are considered to be conservative.

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FOREWORD

This report describes an effort conducted by the University of Dayton Department of Mathematics as a subcontractor to Universal Technology Corporation under Air Force contract F33615-82-C-5089. The task title is Statistical Treatment of Aramid/Epoxy Stress-Rupture Data and it is task number 70. The task was initiated by T.J. Reinhart through AFWAL/Materials Lab.

The work reported herein was performed during the period of 15 Feb 1984 to 30 Apr 1984 by the author Dr. Gerald Shaughnessy (University of Dayton). The report was released by the author in August 1984.

The author would like to thank R.E. Glaser of Lawrence Livermore Laboratory for providing the computer program and output which made the calculations in this report possible.

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1.0 INTRODUCTION

In the report (4) by G. Shaughnessy and J. Snide, the experimental stress-rupture data on S-glass/epoxy of T. T. Chiao (1) was summarized into tabular form which then was incorporated into the appropriate AFSC design manuals. This report will do a similar summary of the stress-rupture data on Aramid/epoxy composite strand data collected by T.T. Chiao and his associates at the Lawrence Livermore National Laboratories. As in (4), this report will provide tables of estimates of quantile levels of the stress-rupture life of strands subjected to sustained tensile loading. The form of the data that was collected is given in Table 1.

TABLE 1

SUMMARY OF STRESS-RUPTURE DATA
FOR ARAMID/EPOXY STRANDS

PERCENTAGE OF FAILURE STRESS	STRESS,KSI	#STRANDS TESTED	#EXACT FAILURE TIMES	#GROUPED FAILURE TIMES	#CENSORED TIMES
90.0	455.2	101	99	2	0
87.0	440.0	100	100	0	0
84.0	424.8	103	103	0	0
80.0	404.6	100	100	0	0
70.0	354.0	49	47	2	0
60.0	303.5	50	41	9	0
50.0	252.9	50	4	0	46

2.0 ANALYSIS

The resulting lifetimes of the stress-rupture tests at each given stress level were modeled as a Weibull distribution. The distribution function for a Weibull distribution is

$$F(t) = 1 - \exp[-(t/\beta)^{\alpha}] \qquad (1)$$

where F(t) is the probability that the lifetime of a given strand subject to a stress (σ) will be less than or equal to t, and α and β are the shape and scale parameters of the Weibull model to be estimated from the experimental data. These parameters, α and β of the Weibull model, vary with the given stress level (σ).

The basic method of analysis used in this paper to estimate the values of α and β of the Weibull model is known as the method of maximum likelihood. In this method, the joint distribution of the observed data (called the likelihood function) is regarded as a function of the parameters α and β. The values $\hat{\alpha}$ and $\hat{\beta}$ that maximized the likelihood function are referred to as maximum likelihood estimates of the parameters α and β. These values can then be substituted into the reliability equations to create the tables.

As stated earlier, the parameters α and β vary with each stress level. The following two relationships were used to describe how α and β varied with the stress level for the data being analyzed.

$$\frac{1}{\alpha(\sigma)} = \theta_1 + \theta_2\sigma + \theta_3\sigma^2 \qquad (2)$$

$$\ln(\beta(\sigma)) = \theta_4 + \theta_5\sigma + \theta_6\sigma^2 + \theta_7\sigma^3 \qquad (3)$$

θ_n are constant coefficents that are to be determined. Other representations of the relationship between α and β and the stress level (σ) were considered but the quadratic model for α and cubic model for β fit best with the experimental data [2].

Using these representations of $\alpha(\sigma)$ and $\beta(\sigma)$, Eqs. 3 and 4 were then substituted into the likelihood equation which was then maximized as a function of the seven variables ($\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7$).

The maximum likelihood estimates ($\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6, \hat{\theta}_7$) are then obtained by setting seven partial derivatives of the likelihood equations equal to zero and solving the resulting seven equations and seven unknowns.

Once the parameters α and β of the Weibull model have been estimated by the above procedure, quantile points in the lifetime distribution of the Aramid/Epoxy composite can be estimated. A quantile is a point in the distribution that has a given percentage of the distribution less than it. For example, the 0.1 quantile in a lifetime distribution is a time such that the probability of a strand failing before that time is 0.1. Using the maximum likelihood estimates ($\hat{\alpha}$ and $\hat{\beta}$), the pth quantile in the lifetime distribution is obtained by setting

$$p = 1 - \exp[-(\frac{t}{\hat{\beta}})^{\hat{\alpha}}] \tag{5}$$

and solving for t. The solution, yielding the pth quantile is:

$$p = \exp[\frac{1}{\hat{\alpha}} \ln(-\ln(1 - p)) + \ln(\hat{\beta})] \tag{6}$$

3.0 Results

Table 2 was generated from Eq. 7 and gives the maximum likelihood estimated life (MLEL) for failure probabilities ranging from 0.0001 to 0.1 and for stress levels ranging from 26% to 45% of the ultimate flow stress (UTS).

For example, at a stress level of 30% (UTS) and a quantile value of 0.001, the table value is 24.6 years. This means that the maximum likelihood estimate of the point in the distribution of lifetimes under which there is a probability of failure of 0.001 is 24.6 years. This estimate has uncertainty in it since it is based on estimates of α and β obtained from the experimental data. Tables 3 and 4 give lower confidence bounds at 95% and 99% levels of confidence, respectively, for quantiles in the life time distribution under which the probability of a strand failing is either 0.0001, 0.001, 0.01, or 0.1. The second set of tables are more conservative and reflect the size of possible errors in the estimation procedures. At the 95% level of confidence, the values in the table were constructed so that there is a probability of 0.95 that the true quantile points in the lifetime distribution are not lower than the table values. To illustrate further the use of the tables, suppose that an estimate of the 0.001 quantile point in the lifetime distribution of a strand subject at a stress of 30% (UTS) is desired. This is a point in the distribution below which there is a probability of 0.001 that a strand will fail. Using Table 2 for the MLEL, the estimate is found to be 24.6 years. Using Table 3 for the lower 95% confidence bound, it is found that there is only a 5% chance that this 0.001 quantile is below 3.5 years.

4.0 CONCLUSIONS

In using the tables in this report for design of pressure vessels for Air Force application, it should be remembered that the analysis was based solely on strand data. Also, that all the estimates are based on extrapolation from the experimental data. No failure data was observed below 50% UTS. Extrapolated estimates from the data have a large amount of variability in them. This large variability resulted in lower confidence bounds for quantiles below 0.01 that are of little value since the variability of the estimates are so large.

There is evidence that the strand data is conservative for design of pressure vessels [3]. The strands tested were weakened by the presence of fluorescent-lights which could be controlled in application. Also, stress-rupture data on vessels have indicated that strand data is conservative in predicting vessel strength [3].

TABLE 2

ESTIMATED LIFETIME OF ARAMID/EPOXY COMPOSITE
UNDER SUSTAINED TENSILE LOADING

MLEL CALCULATIONS (YEARS)
(MAXIMUM LIKELIHOOD ESTIMATED LIFE)

		QUANTILE LEVEL			
%FIBER STRESS	STRESS (KSI)	0.0001 MLEL (YRS)	0.001 MLEL (YRS)	0.01 MLEL (YRS)	0.1 MLEL(YRS)
26	131.5	20.9	44.5	95.1	205.8
27	136.6	18.5	37.8	77.1	159.4
28	141.6	16.6	32.4	63.4	125.5
29	146.7	15.0	28.1	52.8	100.4
30	151.7	13.6	24.6	44.5	81.5
31	156.8	12.5	21.8	38.0	67.1
32	161.9	11.5	19.4	32.8	55.9
33	166.9	10.6	17.4	28.5	47.2
34	172.0	9.8	15.7	25.0	40.3
35	177.0	9.1	14.2	22.2	34.3
36	182.1	8.5	13.0	19.8	30.4
37	187.1	7.9	11.9	17.7	26.7
38	192.2	7.4	10.9	16.0	23.7
39	197.3	6.9	10.0	14.5	21.2
40	202.3	6.5	9.2	13.2	19.0
41	207.4	6.0	8.5	12.1	17.2
42	212.4	5.6	7.8	11.0	15.6
43	217.5	5.1	7.2	10.1	14.3
44	222.6	4.7	6.6	9.3	13.1
45	227.6	4.3	6.1	8.5	12.0

TABLE 3

ESTIMATED LIFETIME OF ARAMID/EPOXY COMPOSITE
UNDER SUSTAINED TENSILE LOADING

LOWER BOUND CALCULATIONS (YEARS)
95% CONFIDENCE INTERVAL

		QUANTILE LEVEL			
%FIBER STRESS	STRESS (KSI)	0.0001 LB (YRS)	0.001 LB (YRS)	0.01 LB (YRS)	0.1 LB (YRS)
26	131.5	1.9	3.5	10.4	24.9
27	136.6	1.9	3.5	9.9	22.6
28	141.6	1.9	3.5	9.4	20.7
29	146.7	1.9	3.5	8.9	19.1
30	151.7	1.9	3.5	8.6	17.7
31	156.8	1.9	3.5	8.3	16.6
32	161.9	1.9	3.5	8.0	15.6
33	166.9	1.9	3.5	7.8	14.7
34	172.0	1.9	3.5	7.6	14.0
35	177.0	1.9	3.5	7.4	13.4
36	182.1	1.9	3.5	7.3	12.8
37	187.1	1.9	3.5	7.1	12.3
38	192.2	1.9	3.5	7.0	11.8
39	197.3	1.9	3.5	6.8	11.3
40	202.3	1.9	3.5	6.7	10.9
41	207.4	1.9	3.5	6.5	10.5
42	212.4	1.9	3.5	6.3	10.1
43	217.5	1.9	3.5	6.1	9.7
44	222.6	1.9	3.5	5.9	9.3
45	227.6	1.9	3.5	5.7	8.8

TABLE 4

ESTIMATED LIFETIME OF ARAMID/EPOXY COMPOSITE
UNDER SUSTAINED TENSILE LOADING

LOWER BOUND CALCULATIONS (YEARS)
99% CONFIDENCE INTERVAL

		QUANTILE LEVEL			
%FIBER STRESS	STRESS (KSI)	0.0001 LB (YRS)	0.001 LB (YRS)	0.01 LB (YRS)	0.1 LB (YRS)
26	131.5	1.4	2.6	5.0	10.4
27	136.6	1.4	2.6	5.0	10.1
28	141.6	1.4	2.6	5.0	9.8
29	146.7	1.4	2.6	5.0	9.6
30	151.7	1.4	2.6	5.0	9.4
31	156.8	1.4	2.6	5.0	9.3
32	161.9	1.4	2.6	5.0	9.2
33	166.9	1.4	2.6	5.0	9.1
34	172.0	1.4	2.6	5.0	9.0
35	177.0	1.4	2.6	5.0	9.0
36	182.1	1.4	2.6	5.0	8.9
37	187.1	1.4	2.6	5.0	8.9
38	192.2	1.4	2.6	5.0	8.8
39	197.3	1.4	2.6	5.0	8.7
40	202.3	1.4	2.6	5.0	8.7
41	207.4	1.4	2.6	5.0	8.6
42	212.4	1.4	2.6	5.0	8.4
43	217.5	1.4	2.6	5.0	8.3
44	222.6	1.4	2.6	4.9	8.1
45	227.6	1.4	2.6	4.8	7.8

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LIST OF SYMBOLS, ABBREVIATIONS, ACRONYMS

MLEL	Maximum likelihood estimated life.
F(t)	Probability that the lifetime of a given strand subject to a stress will be less than or equal to the time the stress is applied.
t	Time.
alpha	Shape parameter from Weibull model.
beta	Scale parameter from Weibull model.
sigma	Stress.
theta1-theta7	Constants.
p	A given probability F(t).
UTS	Ultimate flow stress.
ksi	Kips per square inch.

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